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Chapter 8

Allometry: Size and its consequences, or…
Why aren't there 20 ft tall ants?

Susan M. Schenk

Joint Science Department
The Claremont Colleges
925 N. Mills Ave
Claremont, CA 91711
sschenk@jsd.claremont.edu

Susan Schenk received her BA in Botany from the University of California at Los Angeles and her Ph.D. in Plant Physiology from the University of London, Bedford College. As Lab Instructor and Coordinator in Biology for the Joint Science Department, she designs and teaches labs for the Introductory Biology course. She has been a member of ABLE since 1993 and has presented two mini-workshops in addition to the major workshop included in this volume. Her research interests are in pollination biology and the ecology of coastal sage scrub at the Colleges' Bernard Field Station.

Contents

Introduction

This exercise is part of our introductory level biology course and is intended to introduce the students to the relationships between size, shape, and function in living organisms. Most students know that such relationships are important in chemistry and engineering, but often fail to understand their relevance in biology. Lab setup is straightforward and does not require any expensive materials. Data collection is simple and takes about two hours. The report is fairly long and requires that the students make a number of graphs. In the first two weeks of the term, our students complete a homework exercise (available upon request) that prepares them for this. The procedures in this exercise could very easily be used by students to design their own investigations.

Student Outline

Introduction

Evolution has resulted in changes in the sizes and forms of organisms. Everything about the biology of an animal, including its physiology, anatomy, and ecology, is influenced by its body size. Frequently there seem to be limits on the size that different organisms can attain, even when larger size might be thought to be evolutionarily advantageous. Often an increase or decrease in size is correlated with a change in proportions. In other words, physics seems to put limits on natural selection. So, why aren't attacks by twenty-foot tall insects a problem in real life? Why are the metabolic rates of mice and elephants different? Why is the shape of a baby animal often different from that of an adult? Why do eagles need larger hunting territories than falcons? Understanding the significance of a particular morphology or interpreting the factors which underlie a particular evolutionary trend involve studying the relationships that exist between size, shape, and function. In this lab you will be introduced to allometry, the study of size and its consequences.

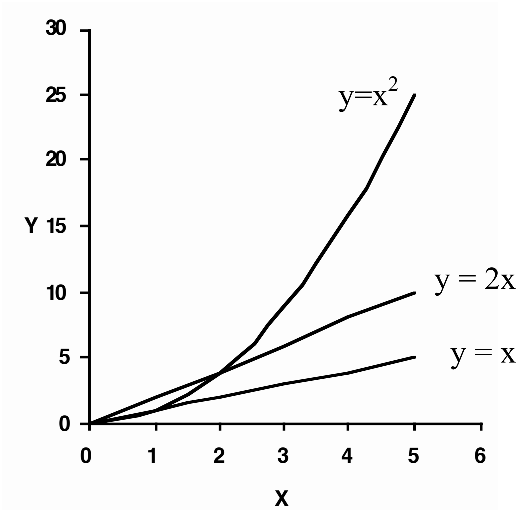
Background

We will be investigating how differences in magnitude in form or function of one variable are correlated with changes in the form or function of another variable. In other words, we are interested in what happens to Y as X varies. Sometimes these relations are simple, but very often biological relationships take the form of a power function.

Review of quantitative relationships

1. Look at the line $y = x$ on the graph below. The equation for this line states that every time X changes, Y changes in the same way that X does. The equation for the line $y = 2x$ states that Y is always double whatever X is. Because the relationship between X and Y in both of these cases is constant, we get a straight line. We can generalize this linear relationship to the form $y = ax$, where "a" can be any number.

2. Now look at the curved line. Here the equation is $y = x^2$. We get a parabola when we graph it because the relationship between X and Y is not constant. We can generalize this relationship with the equation $y = x^Z$.



3. The general allometric equation: Putting the two general equations together gives $y = ax^Z$ which describes the relationship between any two variables. This type of equation is called a power function.

4. *Z* in this equation is called the *scaling factor* and represents how Y changes with X. If $z = 1$, for example, then the relationship is linear and Y and X increase at the same rate. If $z > 1$, then Y increases faster than X. If $z < 0$, then Y is decreasing as X increases. If $0 < z < 1$, then Y is increasing as X increases but doesn't increase as quickly. What is *z* when Y remains constant?

Determining an allometric equation:

Suppose we collect some biological data and suspect there is an allometric (power function) relationship. How do we find *a* and *z*?

1. **Log-transformation of equations:**

First we convert the allometric equation to one that is easier to analyze--a straight line. The formula for a straight line is $y = mx + b$, where m = slope (change in Y over change in X, or "rise-over-run"), and b is the y-intercept (point where the line crosses the Y-axis).

Can we change $y = ax^Z$ into this form? Yes, indeed, if we use logarithms. Recall that a logarithm in base 10 is the power to which 10 must be raised to produce a given number. For instance, the logarithm of 10 is 1 since $10^1 = 10$, the logarithm of 100 is 2 since $10^2 = 100$ and so on.

Recall also that $\log ab = \log a + \log b$
and that $\log a^b = b \log a$

Applying these rules, we log-transform $y = ax^Z$ to get $\log y = z \log x + \log a$

This is in the form of a line with y = log y, x = log x, m = z and b = log a

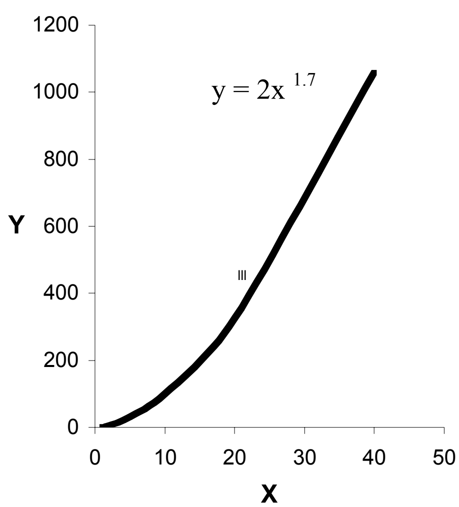
2. **Comparison of a power equation and its log form:**
We can illustrate how conversion to logs helps us by graphing the equation $y = 2x^{1.7}$**.** First we make a table like the one below. To plot the untransformed equation, we make a graph with the axes labeled x and y. We number the x-axis from 0 to 40 and the y-axis from 0 to 1200 and plot the points. As expected, it is a parabola.

Now we make another graph for the log-transformed equation, log y = 1.7 log x + log 2. We calculate the log x and log y values first. We label the axes "log x" and "log y" and number them from 0 to 4. We plot that data and, *voila*, a straight line!

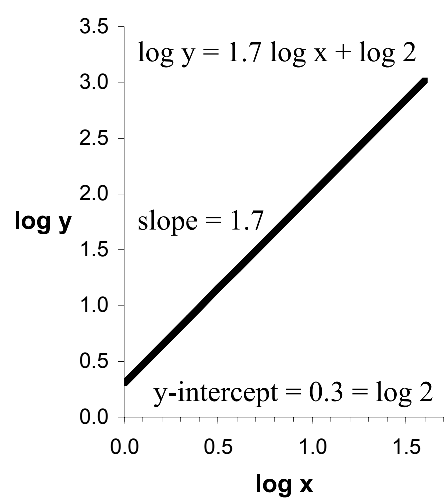
*Note that the slope of the line = z = 1.7 and the y-intercept = log a = 0.3.
The antilog of 0.3 = 2.

For the equation $y = 2x^{1.7}$:

x	x1.7	2x1.7	log x	1.7 log x + log 2
1	1.0	2.0	0.0	0.30
2	3.3	6.6	0.3	0.81
5	15.4	30.8	0.7	1.49
10	50.1	100.2	1.0	2.00
20	162.8	325.6	1.3	2.51
40	529.0	1058.0	1.6	3.02



Power function

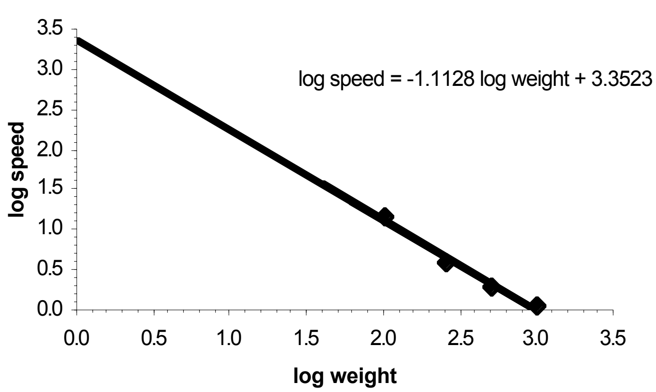


Log-transformed equation

3. **Example of how to find an allometric equation:**
   a) Suppose we suspect there is an allometric (power function) relationship between weight and speed in dragons. We can plot the logs of the data we collect and see if they fall on a relatively straight line. They do, so we have an allometric relationship.
   b) We add the trend line. We use this to get the log-transformed equation, and then use the equation to determine the slope (= *z*) and the antilog of the y-intercept (= *a)*.
   c) We then plug these numbers into the general allometric equation to get the equation that embodies the relationship between the characters.
   d) Once we have this equation, we can use it to make predictions about weight and speed in different dragons and to help us understand the reasons behind the relationship.

Speed vs weight in dragons:

x (kg)	log x	y(km/hr)	log y
100	2.00	15.5	1.19
250	2.40	4.3	0.60
500	2.70	2.1	0.30
1000	3.00	1.2	0.08



We can use Excel to plot the set of log-transformed points and add a trendline. We can then change x to log weight and y to log speed, as at left. For this example, the slope (= z, scaling factor) of the allometric equation is -1.1128 and the y-intercept (= a) is the antilog of 3.3523 (= 2250.6). Substituting these into $y = ax^Z$, the allometric equation is

$$\mathbf{y = 2250.6\ x^{-1.1128}}$$

You can use the equation to calculate the weight of a dragon that can fly at 8 km/hr by replacing y with 8 and solving for x. You can use it to calculate the speed of a 600 kg dragon by replacing x with 600 and solving for y.

There are some limitations to allometric equations. Keep these in mind.

1. An allometric equation *only describes* the relationship between two characters; it says nothing about the reasons for the pattern. It is a starting point, but understanding only comes from a knowledge of the system.
2. The equation may change as the organism grows.

Investigations

Isometry: a special case of allometry

When shape doesn't change as size changes:

Every sphere has the same shape no matter what the size. So does every cube. This type of relationship is a special form of allometry, termed isometry, and it occurs when the objects are geometrically similar. For isometric objects, the ratio of corresponding lengths (L) is some constant, C. The ratio of corresponding areas would of course be C^2 and of corresponding volumes would be C^3. Isometry is rare in living systems.

For isometric objects: width α length^1 (z = 1)
area α length^2 (z = 2)
volume (or weight) α length^3 (z = 3)

The relationship of surface area and volume.

1. What about the ratio of surface area to volume in a set of isometric objects? Is there a constant factor here? Does this stay the same as the objects increase in size?

   Well, area is proportional to some length squared (L^2) and volume to some length cubed (L^3), so the SA/V ratio is proportional to L^2/L^3, which reduces to 1/L. Since L can change, clearly this is not a constant. The surface to volume ratio is inversely proportional to size (length) so it will change as the size of the object changes.

   This simple fact has profound biological significance because organisms interact with their environment through their surfaces.

   For a given shape, volume increases more quickly than surface area, and eventually a point will be reached where the movement of materials through the surface cannot keep pace with the needs of the increased number of cells. Further growth in size then requires a change in shape to increase surface area relative to volume.

2. We will investigate how the ratio of surface area to volume changes if an organism changes size but not shape, and how changing shape affects this ratio, by examining two sets of model "organisms," beakers and graduated cylinders. Note: although beakers are, of course, cylinders too, throughout this exercise “cylinder” will refer to graduated cylinders.

   To begin, let us predict what the scaling factor should be for surface area to volume if the relationship is isometric.

   first, $SA \; \alpha \; L^2$ and $V \; \alpha \; L^3$
   so $L \; \alpha \; SA^{1/2}$ and $L \; \alpha \; V^{1/3}$
   therefore, $SA^{1/2} \; \alpha \; V^{1/3}$
   and $SA \; \alpha \; V^{2/3}$

   So we expect the scaling factor of the power equation relating change in surface area to change in volume to be 2/3 (or 0.67).

Half the lab groups will measure the volume of a 2000 ml beaker (measure from the bottom to the

2000 ml line for height (if the beaker only has markings to 1800, measure to 2.5 cm below the top edge). Add the class mean for V to the table below. The instructor will demonstrate how to use the values in the table to determine the allometric equation for beakers.

Beakers		SA in cm^2					V in cm^3			
Label size		250	600	1000	2000		250	600	1000	2000
Mean		201	361	572	912		190	470	970	
Log of mean		2.30	2.56	2.78	2.96		2.28	2.67	2.99	

The other half of the groups will find SA for the 2000 ml cylinder and add the class mean to the table. Measure the height up to the 2000 ml line. Use the tables for beakers and cylinders to answer the writeup questions.

Cylinders		SA in cm^2					V in cm^3			
Label size		250	500	1000	2000		250	500	1000	2000
Mean		321	459	733			250	500	1000	2000
Log of mean		2.51	2.66	2.86			2.40	2.70	3.00	3.30

Allometry in real life

Now you will look for allometry in two real organisms. Each group will collect data for both celery and for woodlice. Each student should copy the data onto the sheets provided and use it to determine the allometric equations between the given measurements, and then interpret the information in biological terms.

Celery

If we wish to examine the size relationships between genetically identical specimens at different stages of growth, we can't do better than a bunch of celery! Each bunch is a single plant with the youngest stalks (leaves) in the middle. Each stalk shows one or more areas of constriction. Are all the stalks the same shape, regardless of size?

Dismember the bunch of celery and choose eight stalks evenly covering the range of sizes from about 30 mm to 300 mm (not counting the leafy bit at the top). Don't choose ones that are too small to handle or ones without at least one constriction visible. Use a razor blade to cut the top off at the first constriction. Everyone in the group should help do the following for the eight stalks of celery (share the fun of counting all those little squares for the surface area!):

1. Weigh the trimmed stalk.
2. Use a ruler (or measuring tape if the stalk is very curved) to measure the length of the stalk along the outside. Measure to the nearest half mm.
3. Cut the stalk across exactly halfway between the bottom and the first node and carefully measure the maximum width of the crescent-shaped surface to the nearest half mm.
4. Cut a thin slice of the crescent-shaped surface, blot it dry, and put it on a piece of mm graph

paper. Trace around it and count the number of mm squares to determine the area.

5. Use a calculator to determine the logs for all the data collected.

Woodlice

Although some people view them as pests, woodlice (sowbugs) are actually valuable decomposers and rarely destroy healthy plants. They have inspired recipes ("woodlouse sauce", Holt 1885), poetry (Sir John Betjeman), paintings (Paul Klee), novels (John Paul Sartre) and even sermons (Thornton, 1889). There was a time when people kept them in a bag around their necks and popped a few to combat stomach distress!

Woodlice are arthropods and one of the few land-living members of the phylum Crustacea. Like crabs and shrimp, they have exoskeletons and must molt in order to increase in size. Unfortunately, they did not evolve a water-resistant wax for their cuticle like insects did, so they are prone to dessication and are consequently found in dark, damp places. They revel in compost heaps.

Woodlice have sharply-angled antennae and compound eyes, like spiders. There are four pairs of mouthparts, seven main trunk segments (each with one pair of walking legs), then five smaller segments (pleopods) with limbs modified into gills, and finally, the uropods, which project from the rear of the animal and are sensory and defensive in function.

Woodlice reproduce May through September. First, the male crawls across the back of a female and transfers sperm into the genital opening at the base of the pleopods. Eggs are then laid in a fluid-filled pouch, or marsupium, which develops on the underside of the female. The eggs hatch in a few days. The young stay in the marsupium for a few hours and absorb the fluid. At first, the juveniles only have 6 pairs of legs. The seventh pair appears after the first molt. Woodlice molt every few weeks as long as they live, which can be for several years.

We have established cultures that should provide you with a range of sizes of these animals to use in investigating the relations between length, mass, and cooling rate. The rate at which a body heats up or cools down is related to size, and has implications for survival. Sometimes a change in shape is necessary for temperatures to be maintained within tolerable limits. For example, as the volume of an organism increases, more and more metabolic heat is generated. If surface area does not increase sufficiently to allow enough of this heat to be transferred to the air, the organism may die from overheating. You will be investigating the relationship between body size and cooling rate for your woodlice, as well as relationships between lengths and mass.

Important: Treat the woodlice very gently at all times!

1. Use a spoon and a brush very gently to remove eight woodlice (covering the range from about 3mm to 15 mm long, about 0.01g to 0.15g) from the cultures and put them, along with any unavoidable bits of dirt, into a plastic container. Once the woodlice are selected, the group should work together to record the following data for the woodlice.
2. Put a small petri dish bottom on the balance and tare it. Let one of the woodlice crawl onto the spoon and carefully add it to the petri dish. Record the weight and put the lid on the dish. Number the dish. Do the same for all the animals. Take them back to your bench.
3. Put ice into a plastic container and push a 250-ml beaker into it so there is ice below it and halfway up its sides. Let it chill for several minutes.
4. Carefully add the largest woodlouse by inserting the petri dish and tipping the animal out (don't let it drop very far). Immediately start the timer. Gently use the brush to turn the

animal over if it lands on its back. Cover the beaker with a large petri dish lid so that you do not warm the animal up with your breath.

5. The animal will walk around for a short while. If it stops walking, gently prod its back end with a brush. Record the time when this prodding fails to get it walking again. It is now too cold to walk. (Waving antennae doesn't count as walking).
6. Remove the beaker and gently tip the animal out onto a piece of paper. Measure its body length (not including antennae or uropods) and width to the nearest half mm. Be quick–it won't take long to warm up. If it moves before you can measure it, cool it down again.
7. Repeat the process with the other woodlice, letting the beaker chill for 2 or 3 minutes before you add the next animal and making sure that the inside of the beaker is dry.
8. Put the data for all eight of your group's animals in your table.
9. Use a calculator to determine the logs for all the data you have recorded.

Writeup: must be typed, single-spaced

- Answer the questions in the order asked and number the answers.
- The trendlines are actually log equations, so replace x with log (of whatever x represents) and y with log (of whatever y represents). For example, log Wt = 3.2 log L + 5.6.
- Both axes on all graphs should start at 0. Label axes "log Volume" etc.
- Import the graphs into your text document and give them legends.
- When you report allometric equations, use the name or abbreviation of the actual parameter and use superscripts for z. For example, write weight = 3.4 length $^{2.87}$ rather than y = 3.4 x^2.87.
- As the last page of the report, attach your handwritten tables with the data and their logs.

SA to V ratios for beakers and cylinders. Put all of question1 on page 1 of the report.

1. *Comparing beakers and graduated cylinders*:
   Compare the general shape of beakers to that of cylinders.
   What volume has the largest SA/V ratio for beakers? For cylinders?
   If you have two cylindrical objects that hold 1000ml, but one is short and wide and the other is tall and thin, which has the larger surface area?
2. *Figures 1 and 2*: Use Excel to plot log SA (vertical axis) vs log V (horizontal axis) for beakers and for cylinders. Include the trendlines and their equations and r^2 values. They are log equations, so replace x with log V and y with log SA in the equations.
3. *For the beakers*:
   What is the allometric equation that corresponds to the log equation? Refer to page 4 for the method of turning a log equation into the related allometric one.
   How close was *z* (the scaling factor) to 0.67 (= 2/3; see page 6)? Is the set of beakers isometrically related? Explain. If not exactly isometric, are they getting wider or taller faster than you would expect for an isometric relationship?
   Use your equation to predict the surface area for a 100 ml beaker. Show work.
4. *For the cylinders:*
   What is the allometric equation that corresponds to the log equation?
   How close is *z* (the scaling factor) to 0.67? Is the set of cylinders isometrically related? If not

exactly isometrically related, are they closer to isometry than the beakers, or further away from it? Are they getting wider or taller faster than you would expect for an isometric relationship? Since they are man-made, what practical reason can you suggest for this?

Remember: As celery and woodlice grow, length, width and weight will all increase. You need to determine if the increase is isometric and, if not, how the relationships are changing.

Celery: Pages 2 (graphs) and 3 (answers to questions) of your writeup

1. Plot a graph (with legend) for each of the following and fit trendlines.
   Put the log equation and r^2 value on each graph. Put all three graphs on the same page.

   Figure 3: the log of the length (x-axis) against the log of the cross-section width.
   Figure 4: the log of the length (x-axis) against the log of the area.
   Figure 5: the log of the length (x-axis) against the log of the weight.

2. How well do the trendlines fit the data points (r^2)? Explain what may have caused any outliers.
3. Give the allometric equation for the three comparisons (remember: do not use x and y).
4. Explain what scaling factor would be expected for each relationship if it were isometric.
5. Are any of the relationships isometric? If so, which? For those that are not isometric, which measurement is increasing more quickly than you would expect if the relationship were isometric?
6. What do your equations tell you about how the shape of the celery leaf changes as it increases in size?
7. What advantage might these changes have in terms of balancing energy gain and energy expenditure?

Woodlice: Pages 4 (graphs) and 5 (answers to questions) of your writeup

1. Plot graphs for each of the following and fit trendlines. Put the log equation and r^2 value on each graph. Put all three graphs on the same page.
   (*Moving scales:* If you have negative numbers, you may want to move the scale labels to the other side of the vertical axis or the horizontal one. To do this, just double-click on the axis, choose the patterns tab, and tick "high", then OK.

   Figure 6: the log of the length (x-axis) against the log of the width.
   Figure 7: the log of the length (x-axis) against the log of the weight.
   Figure 8: the log of the weight (x-axis) against the log of the cooling time.

2. How well do the trendlines fit the data points (r^2)? What difficulties did you encounter in collecting the data for each comparison? Why is there variability in the data?
3. Give the allometric equation for each of the three comparisons.
4. Explain what the scaling factor would be for the first two relationships if they were isometric.
5. Does z for length vs. width show a relationship that is isometric? If not, is width changing more quickly or more slowly than it would if it were an isometric relationship? What does this tell you about how the shape changes as the animal grows?
6. Does z for the length vs. weight show a relationship that is isometric? If not, is weight changing more quickly or slowly than you would expect? What could this mean about the change in woodlouse thickness as the animal increases in size? If thickness wasn't affected, what other

factors might be affecting the weight (assume you measured it correctly)?

7. Not surprisingly, both the weight and the time to cool increase together, but what sort of relationship is it? Consider the following possibilities:
   - The cooling time may simply be related to the mass. If so, you might expect a woodlouse that weighs twice as much as a smaller one to take twice as long to cool, with a scaling factor of 1. Is the scaling factor you found close to this?
   - The cooling time may simply be related to surface area. If it is, what do you think z should be and why do you think so? Is the z you found close to this?
   - Would the combined effects of mass and surface area account for your z? Explain.
   - Woodlice are not simple, static objects so, in the real world, the cooling time may be related to a number of characteristics and behaviors in addition to mass and surface area. Think carefully about woodlice anatomy, behavior, and/or the data collection procedure you used. What did you notice that might have affected the time taken for the animal to stop moving? Explain.

Synthesis: Page 6

1. Would you expect a 1500-ml beaker to heat and cool more quickly or more slowly than a 1500-ml cylinder? Explain.
   Relate this to living organisms and suggest how SA/V might be a factor in natural selection and the evolution of a species.
2. Remember that cellular processes (metabolism) generate heat. What can you say about the problems that mice and elephants have in maintaining a constant body temperature? What do you believe would be true of their relative metabolic rates?
   What is one reason why elephants have large ears? Why mice have fur and naked tails? Explain your answers.
3. So, why are there no 20 ft tall ants? (Look at your textbook for info on insect anatomy and physiology)
   - Say an ant is around a 1/4 inch tall. If the species evolves isometrically so that an ant increases in height by a factor of about 1000 (to 20 feet tall), by what factors do its leg length and leg cross-section area increase? By what factor would its weight/volume increase? What difficulty is clearly going to arise?
   - Ants are ectotherms. What problems would this cause for such a large ant?
   - What other physiological/anatomical problems do you think might arise?
   - What behavioral changes might have to go along with such an increase in size?

Materials

For the lab:
At least one computer with Excel to demonstrate how to convert data into an allometric equation.
(2) balances for weighing the celery stalks, accurate to 0.01 g
(2) balances for weighing the woodlice, accurate to 0.001 g
(2) plastic shoeboxes containing damp paper towels and enough woodlice so that each student group can choose a set of eight which evenly covers the size range from 3 to 15 mm. Note: the paper towels should just be moist. Too much water will kill the animals.
(3) 2-liter cylinders
(3) 2-liter beakers (each group gets a beaker or a cylinder)

The following materials are those needed for each group of 4 students.
(1) lab marker
(1) roll colored tape (optional)
(1) pair scissors
(1) calculator with logarithmic functions
(1) 15-cm ruler
(1) 30-cm ruler (preferably flexible, to measure curved stalks) or a tape measure
For celery:
(1) untrimmed bunch of celery
(1) sheet of 1-mm graph paper
(1) single-edged razor blade
For the woodlice:
(2) plastic spoons
(2) small brushes, like those used with *Drosophila*
(1) plastic container in which to collect woodlice. This can be the cheap, disposable kind available in most supermarkets. The square, 20-oz kind work fine.
(8) very small petri dishes to hold woodlice
(1) plastic container to hold ice. Again, disposable ones work fine as long as they are about 6 inches in diameter and 4 inches deep.
(1) 250-ml beaker that will be pushed about 2 inches down into the container of ice.
(1) lid from a 9-cm petri dish to cover the top of the 250-ml beaker
(1) timer or stopwatch

Notes for the instructor

- Check that the values for the beakers and cylinders you own agree with the values in the table as your brands may have slightly different proportions to the ones used here.
- Remind the students how to calculate surface area and volume.
- Stress using the indicated units when taking measurements.
- The celery should be untrimmed or the useful size range will be reduced.
- It is helpful to have diagrams of woodlouse (sowbug, potato bug; *e.g., Porcellio* and *Armadillidium* species) anatomy available. Check the behavior of your local species to be sure that the animals do not roll up or curve so much that measuring length and width is excessively difficult. Woodlice can be found in damp, dark places such as under rocks and leaf litter. I have an inexhaustible supply in my compost heap. They can be kept in loosely covered boxes of

moist leaf litter and dirt, and fed rodent pellets or carrots (they eat paper towels, too). They will die equally quickly if their environment is too dry or too wet. Sort out ones for the students to use ahead of time and put them in boxes with damp paper towels. They will hide in the towels.

- Since the purpose of the lab is to learn how to determine allometric relationships, along with how to interpret results and assess the reliability of data, rather than to get the "right" answer, measurement errors are not of paramount importance.
- Students should have prior experience of using a graphing program in order to answer the writeup questions. Our homework assignment on using Excel is available for the asking.
- It is important that the students do a prelab exercise, so one is included as appendix B.

Acknowledgments

This exercise is based in part on Allometry in Biological Systems by Stephen C. Trombulak in *Tested Studies for Laboratory Teaching, Proc 12th Workshop Conference of the Association for Biology Laboratory Education,* 1990, edited by Corey A. Goldman and Lab 1: Size and Shape in Biology from *The Laboratory Manual for BioSci 184*, 1997, University of Chicago, prepared by Thomas F. Colton and Michael C. LaBarbera.

Appendix A: Data Collection Sheet for Allometry Lab

Note: Be sure to take all measurements in the units listed.

Celery: measured values

Stalk	Length (mm)	X-sect width (mm)	X-sect area (mm^2)	Weight (g)
1				
2				
3				
4				
5				
6				
7				
8				

Celery: logs of values above

Stalk	Length	X-sect width	X-sect area	Weight
1				
2				
3				
4				
5				
6				
7				
8				

Woodlice : measured values

Animal	Weight (g)	Cooling time (sec)	Length (mm)	Width (mm)
1				
2				
3				
4				
5				
6				
7				
8				

Woodlice: logs of values above

Animal	Weight	Cooling time	Length	Width
1				
2				
3				
4				
5				
6				
7				
8				

Appendix B: Sample prelab

1. What is the general allometric equation and what does it describe?
   $y = ax^z$. It describes the relationship between two variables

   What is z called and what does it represent?
   z is the scaling factor and it represents how one variable changes as the other changes.

2. How does shape change as size changes in an isometric relationship?
   Shape does not change as size changes, it remains the same.

   What is a major ecological role of woodlice? *They are important decomposers*

   What type of environmental conditions do they need? *Dark and moist*

   To which phylum do they belong? *Crustacea*

3. Follow through the example for dragons on page 3-4 of the lab exercise.
   How heavy is a dragon that can fly at 8 km/hr? (Show work)

   $y = 2250.6\, x^{-1.1128}$
   if $y = 8$, then $8 = 2250.6\, x^{-1.1128}$
   $x^{-1.1128} = \dfrac{8}{2250.6} = 0.00355$
   $x = (0.00355)^{1/-1.1128} = 159$ kg

   OR:
   $\log 8 = -1.1128 \log x + 3.35$
   $0.900309 - 3.35 = \log x = 2.20$
   $x = 159$ kg

   How fast can a 600 kg dragon fly? (Show work).

   $y = 2250.6\,(600)^{-1/1128}$
   $= (2250.6)\,(0.00081)$
   $= 1.82$ km/hr

   OR:
   $\log y = -1.1128 \log 600 + 3.3523$
   $= 0.2607733$
   $y = 1.82$ km/hr

4. What is the allometric equation corresponding to $\log y = 10.8 \log x + \log 3$?

   $y = 3\, x^{10.8}$

   What log equation represents $y = 25\, x^{4.7}$?

   $\log y = 4.7 \log x + \log 25$ (or 1.398)

5. What 4 physical characteristics will you measure in this lab for
   Celery: *length, width cross-section area, weight*
   Woodlice: *length, width, weight, cooling time*

Appendix C: Further Reading

Bunk, S. 1998. Do energy transport systems shape organisms? The Scientist, 12(24):14. Online at www.the-scientist.com

Hunter, P. 2003. Sizing up nature's denizens. The Scientist, 17(18):18. Online at www.the-scientist.com

Niklas, K. J. 1994. Plant allometry: the scaling of form and process. University of Chicago Press, 395 pages

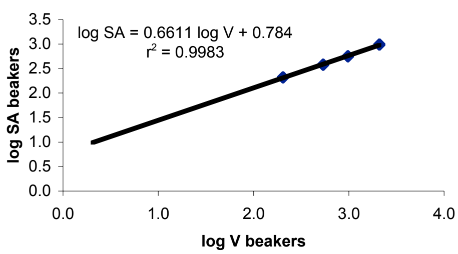
Vogel, S. 1988. Life's Devices: the physical world of plants and animals. Princeton University Press, 367 pages

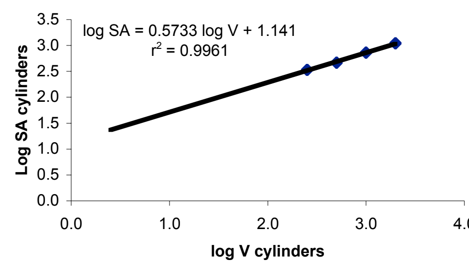
Appendix D: Allometry Writeup Guidelines

Beakers and cylinders

1. In general, the beakers are shorter and fatter than the cylinders.
   The largest SA/V ratios are seen in the smallest beaker and cylinder.
   The tall, thin cylinder would have a greater surface area.

2.





Figures 1 and 2. The relationship between surface area and volume for beakers (left) and graduated cylinders (right).

3. For the beakers: (actual numbers will vary).
   $SA = 6.12\ V^{0.66}$
   z is very close to 0.67, but not exactly isometric. Beakers are getting wider slightly faster than if isometrically related.
   For my data, a 100 ml beaker would have a surface area of about 127 cm^2.

4. For the graduated cylinders: (actual numbers will vary).
   $SA = 13.8\ V^{0.57}$
   z is definitely less than 0.67 and the relationship is not isometric.
   The surface area is not increasing as quickly as expected. Volume is increasing more quickly than expected. They are getting wider faster than they are getting taller. If they stayed in proportion, they would be unstable, and too tall to use easily.

Celery

1.

Figure 3. The relationship between length and width in celery.

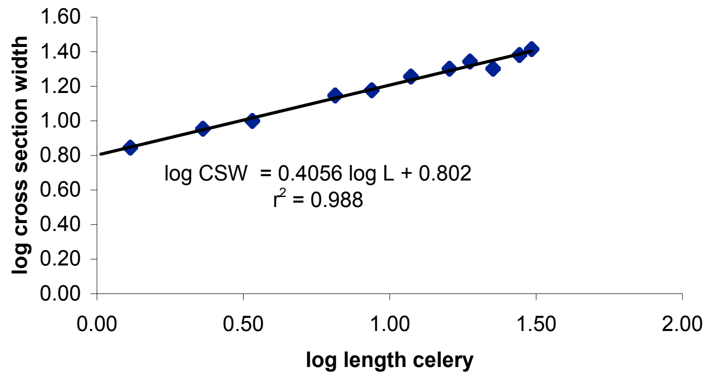


Figure 4.. The relationship between length and cross-section area in celery.

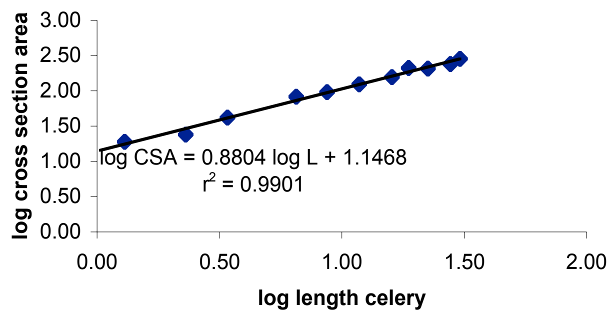
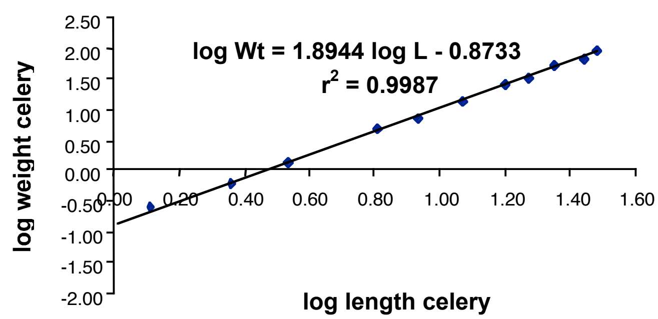


Figure 5. The relationship between length and weight in celery.



2. Trendlines should fit the points well. Students should report the r^2 values. If there are any outliers, it might be an arithmetic error, difficulty in measuring cross-section width or counting squares, or individual variation in the stalks.

3. They will vary depending on the actual data, but should be similar to these:

   Width $=$ 6.3 Length $^{0.41}$
   Area $=$ 14.0 Length $^{0.88}$
   Weight $=$ 7.5 Length $^{1.89}$

4. If the relationships were isometric:
   z for the first equation would be 1, for the second would be 2, and for the third would be 3

5. None of the relationships is isometric.
   In all three comparisons, the length is increasing more quickly.

6. They are getting longer and thinner as they develop.

7. Along these lines: The plant needs to carry out photosynthesis so the more green surface it has to collect light energy the better, but at the same time it must use energy to support the plant body so the less mass compared to photosynthetic surface, the more economical it is. The tight packing of the stalks gives all but the outer ones additional support, and the vascular tissue in the stems also helps support the stalks, so equivalent thickness to short, young stalks is not necessary. In nature, the stalks would not be so tightly packed and more of the inner ones would be green too.

Woodlice

1.

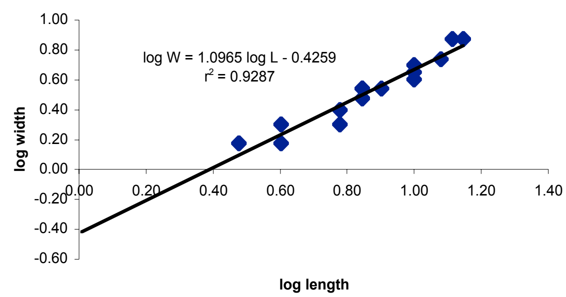


Figure 6. The relationship between length and with in woodlice.

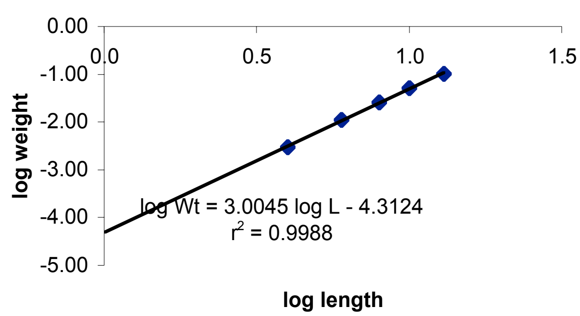
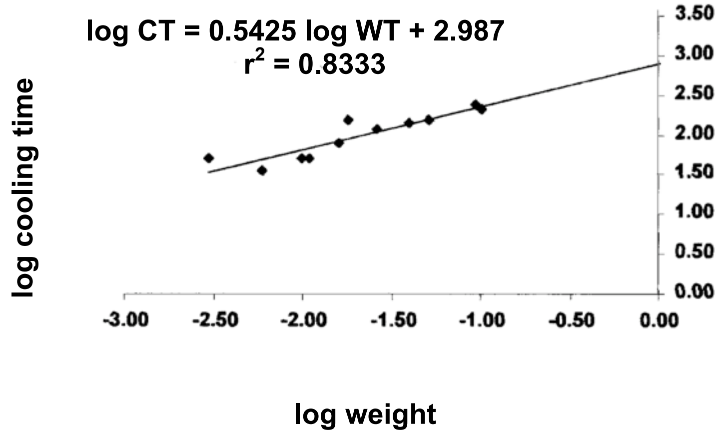


Figure 7. The relationship between length and weight in woodlice

Figure 8. The relationship between weight and cooling time in woodlice.



2. Answers will vary depending on the data collected. Students should refer to the r^2 values. The trendlines fit the points pretty well for length vs width and for length vs weight. Outliers for cooling may be due to difficulty of deciding just when the animal has stopped for good. In all cases, there is natural variation.

3. The allometric equations I got were as follows. The pattern should be reasonably similar for the students, although many get smaller scaling factors for the first two.

   Width = 2.67 Length $^{1.0967}$
   Weight = 0.000048 Length $^{3.0045}$
   Cooling time = 791.9Weight $^{0.5425}$

4. z would be 1 for the first equation and 3 for the second.

5. For the first equation, if z is < 1, the animal is getting relatively longer as size increases. If z>1, the animal is getting relatively wider as size increases.

6. For the second equation, if z<3, then the animal is lighter than expected for isometry as length increases and may be thinner. If z>3, then animals are heavier than isometry would predict and may be thicker. If thickness is not affected, then there may be more empty space under the carapace, legs may make up a different proportion of the weight, the exoskeleton may vary in thickness, internal density may change, etc.

7. Mass: z is unlikely to be close to1

- Surface area: z would be close to 0.67 as for the beakers and cylinders. Mostly way under this, so unlikely. Students with a z that is much different from 3 for length vs weight might talk about the increased/decreased SA/V their value for z indicates.
- If z were between 1 and 0.67, maybe. Since observed z for cooling time is almost always much less than 0.67, it's unlikely.
- The cooling took place mainly from the bottom, with some from the sides of the beaker. Woodlice extend their legs as they chill to distance their bodies from the ice. They also stop moving when their legs are too cold to bend. This stiffening is likely to happen long before the whole mass of the animal is cooled to a uniform temperature. Therefore the measured z is not likely to reflect the actual cooling time for the whole animal and so is not simply related to mass or surface area. Students will most likely suggest other possibilities of greater or lesser likelihood.

Synthesis

3. A beaker would heat and cool more slowly than a similarly-sized cylinder because the cylinder would have a greater surface area compared to its volume and therefore more area per unit volume through which heat can be gained or lost.
   If there are selection pressures leading to a change in size, at some point this may produce a change in shape as well which increases or decreases the SA/V. Example: animals in cold climates may evolve towards smaller SA/V ratios which would help retain heat, and in warm climates ones of similar volume may tend towards higher SA/V ratios which would help release heat. Any reasonable example.

4. Elephants have a much smaller surface area to volume ratio than do mice, so they would lose and gain heat less quickly than would mice. Because they would lose heat more slowly, and much of it would have a farther distance to travel to the surface to be dissipated, the elephants would need to have a slower metabolic rate to prevent overheating. The mice would have a higher metabolic rate to replace lost heat. Elephant's large ears act to radiate excess heat brought to the area by the blood. The fur on mice helps to prevent heat loss, and the naked tail facilitates heat loss.

Along these lines:

- Leg length would increase by 10^3 also; CS will increase by 10^6 and volume/weight by 10^9. This proportionately greater increase in the weight of an isometric twenty foot tall ant without a change in the proportions would cause its legs to collapse under the strain and its head to droop for similar reasons.
- As ectotherms, the temperature of ants mostly mirrors the environment. If the ant is too large, it will not be able to warm itself throughout.
- Something like: The increase in size would require modifications of the nervous system to make sure information was distributed everywhere in a timely fashion. Ants would need far more food and lots more of it or larger food which might require changes in food acquisition and digestion. Circulation of oxygen would need to be modified as well, as would the method of keeping warm.
- Something like: Ants would not be able to feed in the same way, or live in such large groups, or make similar nests.