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T. rex Can't Jump, or could it? A biomechanical inquiry lab

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William Beachly has his Ph.D. from the University of Nebraska where he worked on mathematical models and experimental systems to study foraging behavior in spiders, sunfish, and bluejays. Since 1997 he has been teaching courses in ecology, diversity, invertebrate and vertebrate zoology, human physiology and mathematical biology at Hastings College. His previous ABLE workshops include "Death by osmosis" in 1999 and "Why not make a game of it?" in 2000. He credits many of his laboratory ideas to the challenges of teaching a summer science program to gifted eighth graders.

Introduction

Students in Biology, Anatomy, and Physiology courses struggle with mathematical formulas, but can be encouraged to adopt mathematical reasoning when an engaging problem is posed. This lab challenges students to find out for themselves if a *Tyrannosaurus rex* had the ability to leap (view Charles R. Knights' painting "Leaping Laelops" at www.charlesrknight.com/AMNH.htm.) In doing so students discover 'ways to solve it' in groups, become familiar with the concept of proportionality, apply simple physical principles, and get to know a chicken leg like they never did before. My approach to this lab is to give them handouts with the information they need as they go along, rather than a big packet at first. To do the entire exercise could easily take 3 hours or more, but there are ways to break it up over more than one period or with students working on some parts on their own time. There are also potential shortcuts or extensions to suit the focus of the course. In this paper I will outline the basic steps I follow and illustrate some of the handouts I use, but to reproduce all 13 in their entirety here would take too much space. Instead you may contact me if you would like me to send paper copies of them: wbeachly@hastings.edu.

Procedures

Without reproducing each handout in its entirety, this section will summarize what the students are given and asked to do. I introduce the controversy over how *T. rex* foraged, poll the students to find their opinions, and ask them to share ideas of how we could test a hypothesis of a historic nature (since we obviously can't observe *T. rex* now.)

Fundamentals of Forces

I distribute first a handout explaining what force means, what matter is, SI units and how a Newton (N) is defined. Also the formula F=ma is introduced and the acceleration g (relation of mass and weight.) The difference between pressure (measured in Pascals or N/m^2) and force is also illustrated.

The Moment Rule

I made a simple beam balance is for each group by cutting a wooden meter stick in half and pivoting the midpoint of each half on a nail attached to a short base (Figure1). I also provide some brass weights (50g and 100g) with paper clips that allow them to be suspended, and slid along, the arms of the balance. I say "Let M_1 and M_2 be the two masses and D_1 and D_2 be the two distances from the nail. Express the Moment Rule as a simple equation." After a bit of experimentation most groups come up with $M_1D_1=M_2D_2$, or its ratio equivalent. Next I illustrate how this applies to all types of levers, including their forearm (Figure 2).

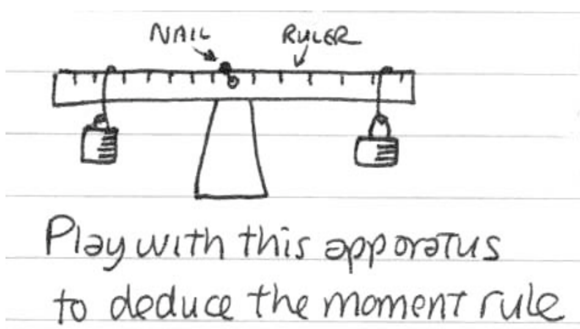


Figure 1. Simple beam balance

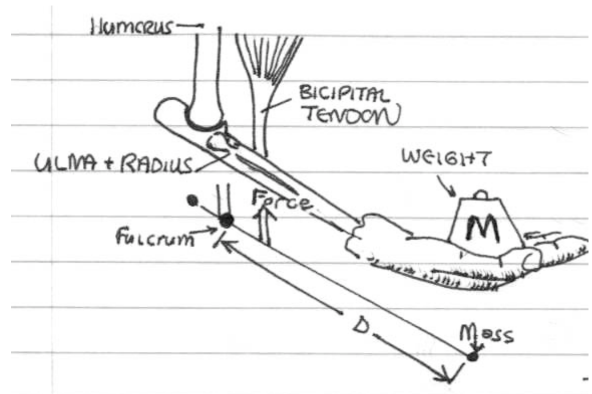


Figure 2. Forearm as a lever

I ask the students: "If D is 35 cm and M is 1 kg and the insertion of the tendon is 35 mm from the fulcrum, what force must be applied by the biceps just to hold the mass of the weight?" The answer, using the moment rule is 10 kg (which is 10 x 9.81 = 98.1 N.)

Area of the Hamstring Muscles

Now the students are shown how, applying proportions, they may estimate the cross-sectional area of the hamstring muscles of a subject chosen within each group. The subject uses a flexible measuring tape to determine the circumference of their leg at mid-thigh. Such tapes with metric units to 150 cm can be purchased inexpensively at craft stores. Then I have the groups calculate the cross sectional area of the leg using the assumption it is circular (they know $2\pi r$, then they find πr^2.)

Next I project an overhead transparency showing a cross section of a cadaver's leg at mid-thigh. Most anatomy texts have figures illustrating this section, but I scanned a photograph on p. 299 of the Color Atlas of Human Anatomy*,* 2nd Ed. (McMinn and Hutchings, 1977) and enlarged it to 8.5 x 11 in. Upon this transparency I lay another transparency photocopied from a piece of graph paper. Students or the instructor may outline the leg and the four principle hamstring muscles (semimembranosus, semitendinosus, long and short heads of biceps femoris) on the graphed transparency (Figure 3). Then I introduce the "spot method" for estimating the area of the hamstrings as a proportion of the total area: "Outline in blocks every square completely within the tracing and put a spot in each square that is more than half within the tracing. Then count the squares within blocks and with spots." They do this for both the entire area of the section and the area of hamstring muscles. Typically these four muscles constitute 20% of the total cross sectional area at this level. The spot method will be applied again later on in this lab.

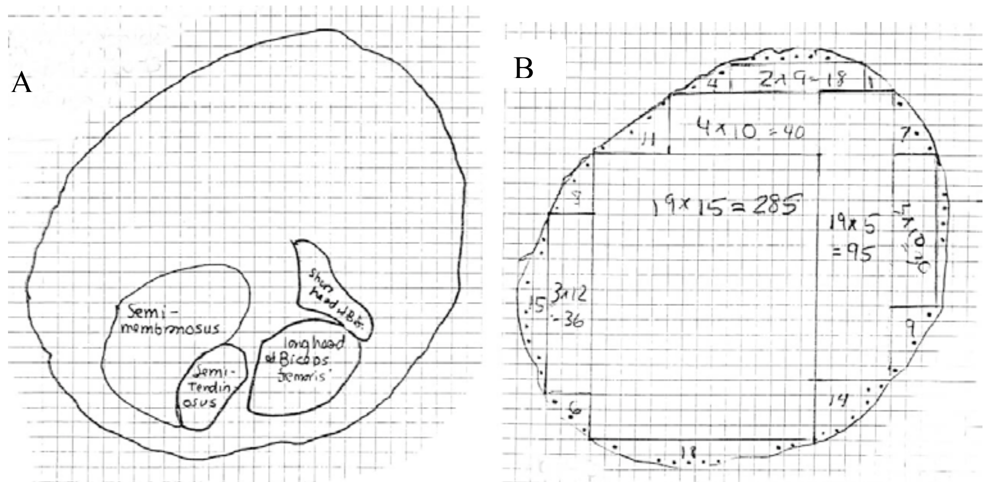


Figure 3. A. Tracing of the hamstring muscles on a mid-thigh section .
B. Using the spot method to estimate the total area.

Measuring the Strength of the Hamstrings

The subject chosen in each group, whose thigh circumference was measured, is seated on a stool next to the apparatus illustrated in Figure 4. I use is a metal bar (55.5 x 5 x 0.5 cm) for the lever with a 1 cm diameter hole centered 2.5 cm from one end (that allows the bar to pivot on a metal rod clamped to the lab bench) and two smaller holes centered 4 cm and 50 cm from the pivot hole. A strong piece of wood lathe or furring strip could be used instead of metal. The 50-cm hole is attached to a 20-Newton Ohaus dial scale (No. 5150) by picture hanging wire. The 4-cm hole is attached by two lengths of picture hanging wire to the ends of an old leather belt, about 90 cm long with metal grommets. The relative distances of 4 cm and 50 cm from the pivot were chosen to reflect the relative distance of the insertion of the hamstrings on the tibia (a' in Figure 4) and the heel (a' + b') from the pivot point of the tibia. The student is seated on a stool with the lower leg held vertical to the ground and at a distance from the apparatus so the picture hanging wire is just taut. Other members of his/her group will stabilize the stool or read the scale. The subject is warned to use only their right leg to pull straight back on the belt, slung around their heel, and not to push back against the lab bench. Each subject does three trials and the average force generated on the scale is noted (usually there will be 2-3 N of initial force at the beginning which is subtracted from the total). I've found most students to be in the range from 10 to (rarely) 20 N. Next the students measure the distances a and b on the bar and are challenged to use the moment rule to find the force applied at the 4-cm hole, and assuming similarly proportioned distances apply within the leg, they calculate the force at the hamstrings insertion.

Calculating the strength of skeletal muscle

Now if the force generated by the hamstrings is known and their cross sectional area is known, students can calculate the force/cm^2. Many physiology textbooks express this in kg/ cm^2 rather than Newtons so students use F=ma to convert units. Many of my students found results within the 3-4 kg/ cm^2 range given in textbooks. Note that this average strength of a muscle depends only on it's area, not its length, though we must give some consideration to the anatomy of the muscle itself.

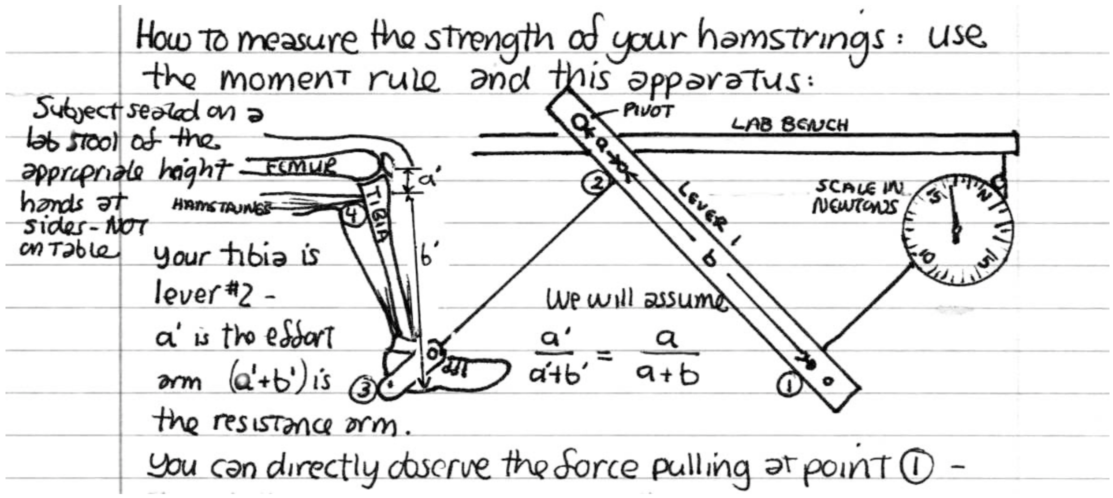


Figure 4. Apparatus used to measure the force generated by a subject's hamstrings

Pinnate vs. parallel muscle fibers

The figure cited for muscle strength in most textbooks is an average for different types of skeletal muscles. In the hamstring group are muscles with some parallel fibers but mostly with unipinnate or bipinnate fibers. In the pinnate muscles fibers converge on a tendon at an angle that deviates from pulling directly in line with the tendon (α in Figure 5). Why does this matter? Consider the analogous situation Dorothy faces in Figure 6:

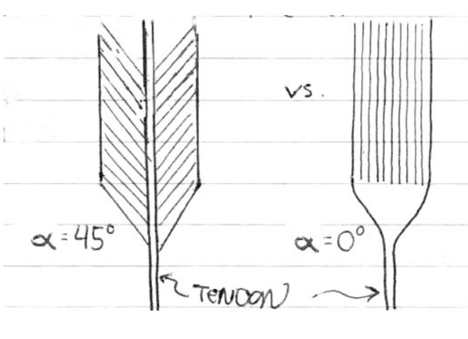


Figure 5. A pinnate vs. parallel muscle has more fibers packed in a given volume, but their angle of incidence (σ) is not optimal.

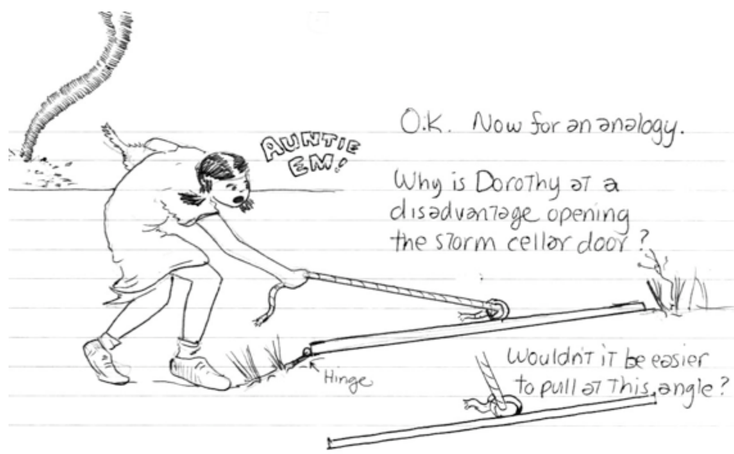


Figure 6. Why the angle matters.

For the very same reason, a tendon that pulls on a bone at an inclined angle will not be optimally placed to move that bone. The bicipital tendon illustrated in Figure 2 would seem to be at an advantage over the "Achilles tendon" of *Tyrannosaurus* in Figure 7. Most mammals have an extended calcaneous that improves the mechanical advantage and angle of incidence of this tendon, but none of the dinosaurs had a "heel". However close attention to the pivot point, near the front of the ankle, suggests that the angle disadvantage is not as severe as it seems at first glance. The nearly vertical tendon pulls up on the nearly horizontal top of the foot bone as a short moment arm extending to the front of the ankle. Figure 7 is redrawn from Dynamics of Dinosaurs and Other Extinct Giants by R. McNeill Alexander, which I highly recommend. Another way to improve on the angle of a tendon is to "jib it up" as we do with our patellar tendon. Our patella, like a jib on a crane, helps to pitch the angle of tendon's insertion to the face of the tibia (Figure 8.)

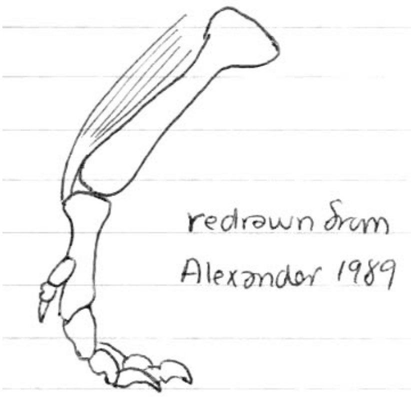


Figure 7. Foot of *T. rex*

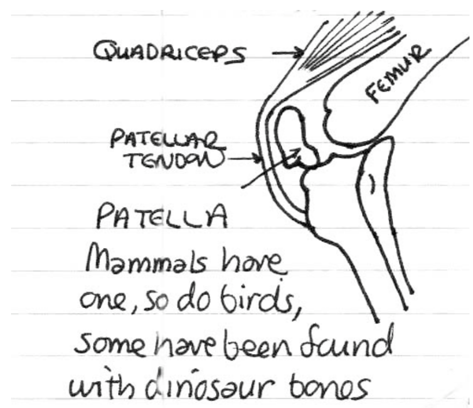


Figure 8. Patella bone in humans

Investigation of the angle of insertion

The "Dorothy example" gives students an intuitive understanding of the effect of angle of insertion, whether of fiber on tendon or tendon on bone. But this simple apparatus allows them to see how trigonometry describes the magnitude of this effect. Cut about 60+ cm from a meter stick so the 10 cm increments are not right at the ends. Drill a 1/4 in. hole through the 3rd 10-cm mark from one end for a pivot point and screw small eyelets into the marks 20 cm from the pivot and in the remaining two 10-cm marks on one side. Hardware stores sell 3-in. wide black suction cups with eyebolts in them. Stick one to a clean blackboard and pivot the ruler on the bolt, then hang two 10 N (1000 g) spring scales (from Ohaus or Pesola) from the 1st and 6th eyelet as shown in Figure 9. As an inquiry-based activity I have students answer this problem: "If the scale on the left reads 10 N and you keep the lever horizontal, what would you predict, using the moment rule, the scale on the right should read?" (answer: 6.7 N) "Now test your prediction with the right scale held perpendicular to the ruler."

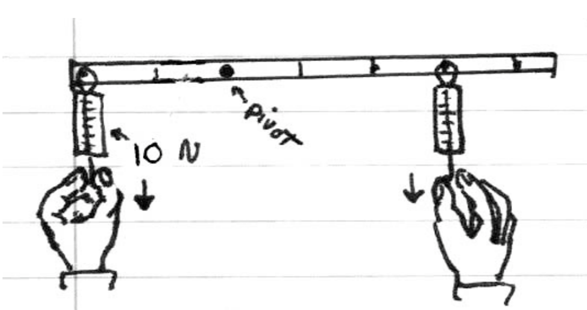


Figure 9: A force applied perpendicular

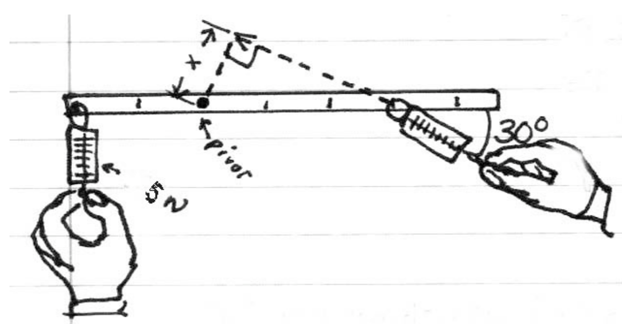


Figure 10: A force applied at an acute angle

Next I have students use a protractor to draw a 30° angle on the board through the 6th eyelet as shown in Figure 10 and then drop a perpendicular line to intercept the pivot. Now they pull with only 5 N on the left scale and determine what it takes on the right scale to maintain the ruler horizontally. "Measuring the length of segment x. Does this distance times the force of the right scale equal the distance times force on the left side? What kind of a triangle is this? What is its smallest angle? What do we call the side along the ruler? What function describes the relationship between side x and the smallest angle?" If students have had trigonometry lately, they will identify the sine function as sin(α)= side opposite/hypotenuse As α approaches 0° the ratio approaches 0 and as α approaches 90° the ratio approaches 1. Thus we can use the sine function to measure the decrement in effective force as the angle of insertion deviates from 90^{o}. This will be very useful.

Scaling up

Since we have no really big bipedal animals handy but us (though we can learn a lot about hopping from kangaroos) and since humans hold their body much differently than the bipedal dinosaurs did, we will seek a smaller model for *T. rex* and scale things up. I illustrate the caution that linear measures, surface measures, and volume measures scale differently by comparing a kiwi to a watermelon. This is the lesson of allometry and we will need to keep this in mind when scaling. There's no reason to expect the force/cm^2 of muscle cross section to be scale dependent, but the mass that this force works on will increase as the cube of any linear measure. Estimates of *T. rex* mass are discussed and justified in Alexander (1989), and a reasonable estimate we'll use is 7000 kg (7 metric tons). We will make two assumptions:

1. *T. rex* 's skeletal muscle generates the same force/cm^2 as ours does.
2. *T. rex* 's leg muscles are similar in proportion to one of its living bipedal relatives.

This living relative is the chicken. Full "leg quarters" are easily obtained at the grocers and are ideal for dissection. I douse them with alcohol (ethyl or isopropyl) to kill bacteria before and after skinning. It is best if you can find "free range" chickens; less greasy fat and *T. rex* was free range too. Figure 11 identifies the major muscles in the leg and Table 1 is a key to the drawing and lists their functions.

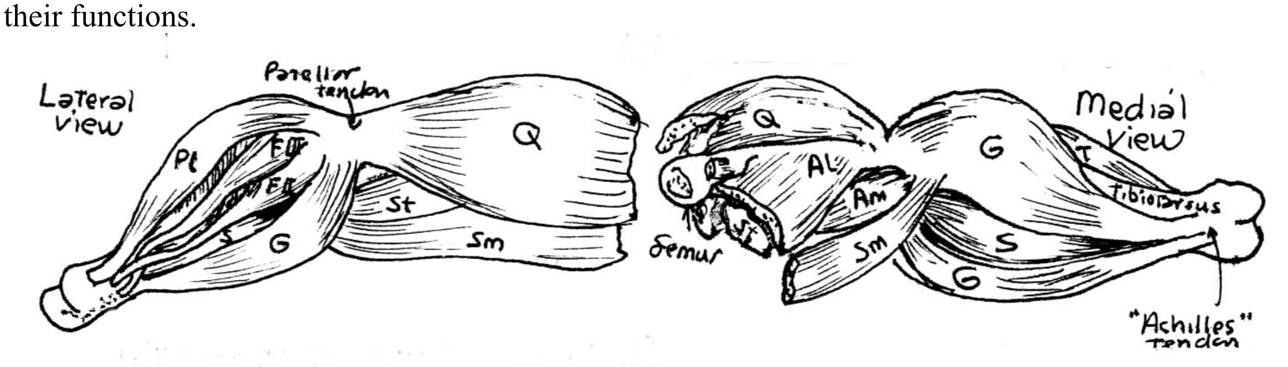


Figure 11. Muscles of the chicken leg (left)

Students need to use blunt dissection to separate muscle bellies and retain origins and insertions; do not allow them to cut through any yet. Fresh specimens allow students to flex and explore muscle actions and in particular identify groups that extend the ankle and knee joints. These will include the quadriceps (all 4 heads) and gastrocnemius (and soleus). These are the largest muscles of the thigh and shank respectively. These muscles would be the prime movers in a leap. The peroneus and flexors of the digits would also assist, as would some hip muscles that are usually not seen on chicken leg quarters. Students may be asked to identify these muscles.

Table 1. Principal muscles in a chicken leg.

Muscle	Insertion	Action and comments
Q = quadriceps femoris	patellar tendon	extends knee, deep to 2 thin muscles
Al = adductor longus	medial face of femur	adducts and extends thigh
Am = ambiens	medial face of femur	flexes the thigh, deep to adductor longus
St = semitendinosus	distal end of femur	extends the thigh back
Sm = semimembranosus	proximal tibiotarsus	flexes the knee
G= gastrocnemius	Achilles tendon	Extends ankle, acts with deeper soleus (**S**)
Pl = peroneus longus	tendon to digits	flexes digits, acts with flexors (**FII**, **FIII**)
T = tibialis anterior	tarsals of foot	flexes ankle

I have the students measure with a tape the greatest girth of the thigh and shank (typically about 2 cm from their proximal ends). At the level measured they cut across the quadriceps to the femur with a razor blade or scalpel. I provide 3x5 cm pieces of a transparency of 2.5 mm graph paper and a permanent fine-point sharpie. They press one side of the transparency grid against the cut edge of the quadriceps and outline it on the other side. They may cut a small hole to accommodate the passage of the bone. Then they repeat this for the ankle and foot extensors, again cutting exactly where they measured the girth. Figure 12 shows photos of this process.

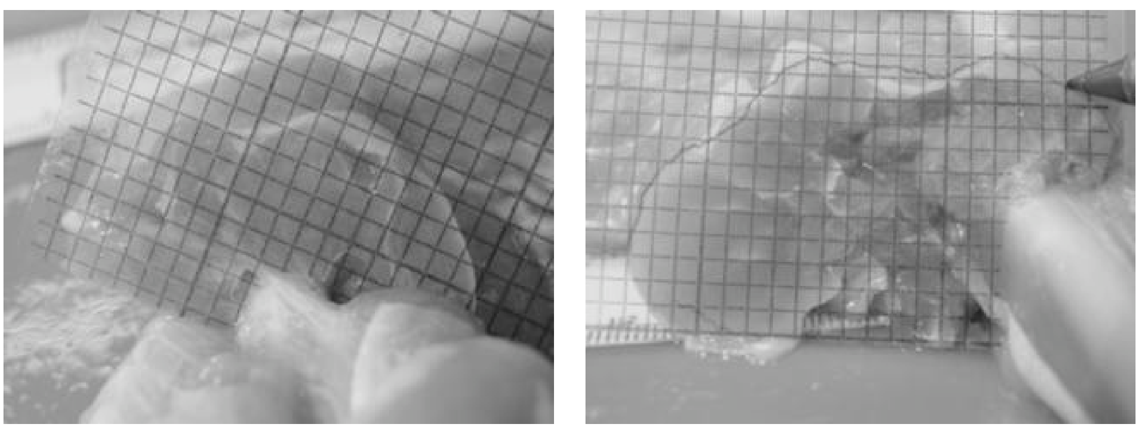


Figure 12. Tracing the cross sectional areas of upper and lower extensor muscles

Now they can use the spot method to measure the cross sectional (c.s.) area of each muscle group on the transparency grid. Each square is 2.5x2.5 mm or 6.25 mm^2. The girths they measured are converted to circular areas, as they did with their human subject's thigh earlier. They calculate the proportion of thigh c.s. area and shank c.s area that these prime movers constitute. The thigh girth is a linear measure and can be scaled directly to another linear measure, the length of the femur. So students find how many femur lengths go into this girth. The shank girth is similarly scaled to tibiotarsus length. On the leg illustrated above the thigh girth was 10.2 cm (=$2\pi r$) so the c.s. area is 8.23 cm^2. The femur length was 7 cm so the girth was 1.46 femur lengths. The shank girth was 11.5 cm (1.28 tibiotarsus lengths @ 9cm) so its c.s. area was 10.55 cm^2.

For this chicken the area of the extensor muscles determined by the spot method was 7.19 cm^2 in the thigh and 9.50 cm^2 in the shank. Students use the concept of proportionality: Thigh extensors = 7.19/8.23 = 0.87 thigh c.s. area. Shank extensors = 9.50./10.55 = 0.90 shank c.s. area.

Fleshing out *T. rex*

How much like a chicken leg did *T. rex*'s leg look? Figure 13 shows how Jack Horner pictures a 'relatively lean' individual based on muscle reconstruction. The resemblance to a drumstick is striking. I measured leg dimensions on a half-scale *T. rex* model made by Wonderworks on display in our local museum: thigh girth=118 cm; thigh length=104 cm; proportional thigh girth=1.13 thigh lengths, shank girth=95cm; shank length=55cm; proportional shank girth=1.73 shank lengths. Thus the shank is relatively shorter and thicker in the dinosaur.

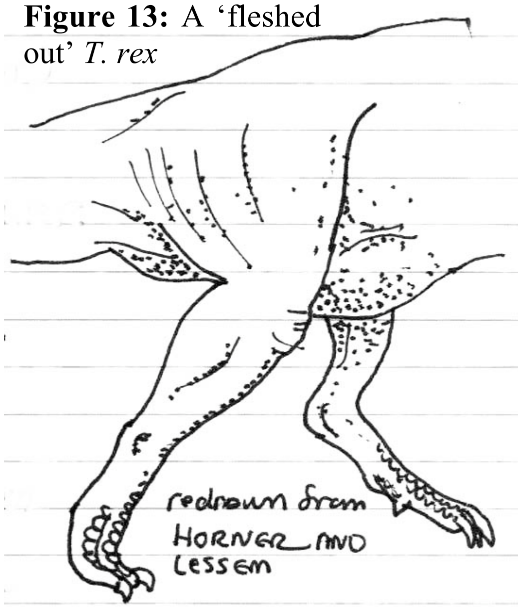


Figure 13: A 'fleshed out' *T. rex*

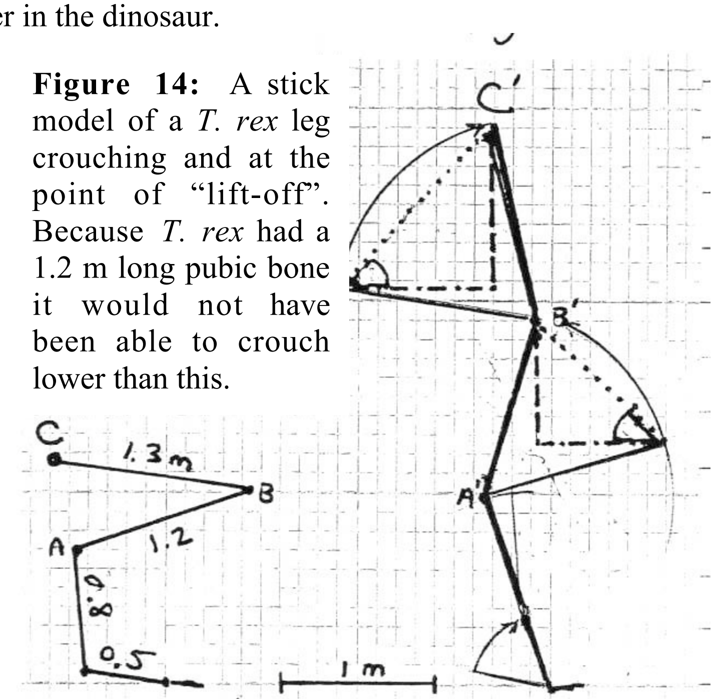


Figure 14: A stick model of a *T. rex* leg crouching and at the point of "lift-off". Because *T. rex* had a 1.2 m long pubic bone it would not have been able to crouch lower than this.

There is a nice drawing of a *T. rex* skeleton in stride on page 130 in The Complete *T. rex* (Horner and Lessem, 1993) which I scaled to a reasonable (for a 7-ton adult) total length of 12 m. At this scale the femur is 1.3 m long and the tibiotarsus is 1.2 m long. The foot sections were also measured and included in the stick figure shown in Figure 14. Arcs of movement are illustrated for points C and B relative to the next most distal joint. The dotted lines (chords) show the displacement vectors of points C and B during the leap. We will attend to these shortly. But first, given the lengths of the femur and tibiotarsus, we can estimate the c.s. area of the extensor muscles (exms) in each as follows (based on the chicken leg I measured, students would use their own derived proportions):

Femur length: 1.3 m	Tibia length 1.2 m
Thigh girth: 1.3 x 1.46 = 1.9 m	Shank girth: 1.2 x 1.28 = 1.5 m
Thigh c.s. : $(1.9/2\pi)^2\pi = 0.29\ m^2$	Shank c.s.: $(1.5/2\pi)^2\pi = 0.18\ m^2$
em c.s.: 0.29 x 0.87 = 0.25 m^2 = 2.5 x $10^3\ cm^2$	em c.s.: 0.18 x .90 = 0.16 m^2 = 1.6 x $10^3\ cm^2$
Applied force: 3.5* kg/ cm^2 x 2.5 x $10^3\ cm^2$	Applied force: 3.5 kg/ cm^2 x 1.6 x $10^3\ cm^2$ = 5.6 x 10^3 kg
(*assume avg. of textbook values) = 8.75 x 10^3 kg	
in Newtons: 8.75 x 10^3 kg x 9.81 m/s^2 = **85.8 x 10^3 N**	5.6 x 10^3 kg x 9.81 m/s^2 = **54.9 x 10^3 N**

Calculating the forces

These would be the forces applied to the patellar and Achilles tendons by the extensors.But these tendons do not have an optimal (perpendicular) angle of insertion on the bones they must move, and while this angle changes during a leap, a reasonable assumption is an average angle of

35°. The sine of 35° is 0.574 and thus we will assume below that the effective force is discounted by that proportion.

But first let us consider the minimum force we need for 'lift off'. We are assuming a 7000 kg mass, so what force would just be enough to counteract the force of gravity? Clearly from F=ma, this will be equal and opposite to F=mg or 7 x 10^3 kg x 9.81 m/s^2 = **68.7 x 10^3 N**.

The only forces that matter in this regard are those directed upwards (we'll ignore the horizontal component of this leap). Vectors show us how to find these forces from our stick diagram (Figure 14) the most important arcs of which are reproduced in Figure 15 below.

Figure 15A: Vectors showing displacement of the hip joint

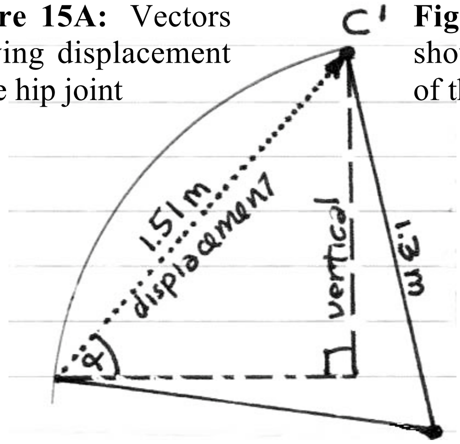
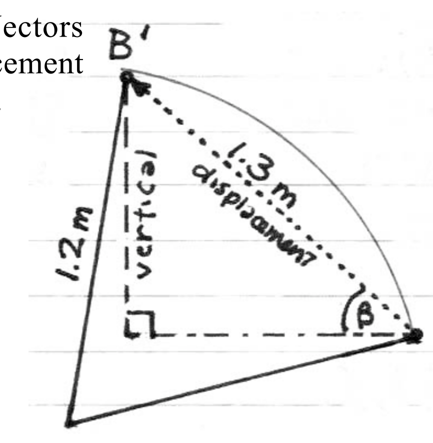


Figure 15B: Vectors showing displacement of the knee joint.



Each vector has a force and distance component. The 1.51 m vector (call ic) was measured directly from the scale on the stick diagram and its force would be that generated by the extensor muscles in the thigh. The 1.3 m vector (call it b) has a force generated by the extensors in the shank. Allowing now for the angle of insertion of the tendons discussed above these forces are:

c: (0.574) 85.8 x 10^3 N = 49.3 x 10^3 N   b: (0.574) 54.9 x 10^3 N = 31.5 x 10^3 N

But recall that only the vertical displacement is relevant to 'lift off' so to find the magnitude of these (dashed) vectors students measure the angles (α and β) and apply the length relations of the sine function (side opposite/hypotenuse) to the magnitudes:

sin α = vertical lift/ force of c	sin β = vertical lift/ force of b
sin 48° = .743 = vertical lift / 49.3 x 10^3 N	sin 43° = .682 = vertical lift / 31.5 x 10^3 N
vertical lift = (0.743) 49.3 x 10^3 N	vertical lift = (0.682) 31.5 x 10^3 N
= 36.6 x 10^3 N	= 21.5 x 10^3 N

Total vertical lift = 36.6 x 10^3 N + 21.5 x 10^3 N = 58.1 x 10^3 N

We see that this falls short of the required force for 'lift off' of 68.7 x 10^3 N. But wait, this is the force generated by just one leg! We must then conclude that 116.2 x 10^3 N is the actual vertical lift. Since we have not considered the role of intrinsic foot muscle or hip extensors (because they are difficult to measure in the chicken legs) our estimate here may be conservative. Could *T. rex* jump? Perhaps our question should be "How high?"

Final calculations and caveats

We can assume that point C in Figure 14 is close to the center of gravity of*T. rex* and based on the scale of our figure, the vertical displacement of C to C' is 2.25 m. During ascent, kinetic energy becomes potential energy in an amount found by: vertical displacement x vertical lift, which would be 2.25 m x 116.2 x 10^3 N = 2.61 x 10^5 N-m. At the apogee of its leap an animal at height h has potential energy only, and equivalent to that which is liberated during its fall which will be mgh. In our example: 2.61 x 10^5 N-m = 7000 kg x 9.81 m/s^2 x h (let students confirm that the units match.) Now we can solve for h: h = 2.61 x 10^5 / 6.87 x 10^4 = 3.8 m. That would be the predicted vertical displacement of C from the crouching position to apogee.

A 3.8 m leap does sound incredible for a 7 ton *T. rex*. But have we been unreasonable in our assumptions? There's no real difference in the strength of skeletal muscle measured in living vertebrates, cold or warm-blooded. Our chicken leg had a relatively longer shank, but its girth was fewer shank-lengths around. If we applied the proportions of the model *T. rex* I measured in a museum to the skeleton shown in Horner and Lessem (1993) the shank girth would be (1.73)1.2 m = 2.08 m for a c.s. area of .35 m^2. If 0.9 of this area is extensor muscles then their area is 3.1 x10^3 cm^2 which is nearly twice that we estimated by scaling up a chicken leg! It would seem our chicken model is a conservative one; more likely to underestimate the muscle power in the leg of a real *T. rex*. Another problem is whether *T. rex* could land from such a height without breaking bones. Alexander (1983, 1989) and Vogel (1988) show ways to consider this. Science News (3/2/02 v. 161:131) has a good article on whether *T. rex* could run.

For this paper I used an average textbook value of 3.5 kg/cm^2 but I strongly encourage students to believe in and use their own estimates; class averages usually are in the 3-4 kg/cm^2 range anyway. Also, I used one chicken leg here as an example but class averages of the proportions are better, and shows the students a rationale (proportions ignore absolute size differences) and practice in using means.

As a follow-up students, especially in comparative anatomy courses, should find references to the on-going debate about *T. rex* and how it foraged. In addition to those I've mentioned the articles by Achenbach (2003) and Ericson (2004) and the book by Bakker (1986) are worth having around.

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