Developing Algebraic and Geometric Understanding of Stereology in Biological and Astronomy Contexts

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INTRODUCTION

The driving principle is to develop innovative biology curricular investigations that integrate biology and mathematics for science classes at the middle and secondary levels, and introductory college courses. Here we outlined an approach to modeling objects such as biological tumors or small objects observed in electron photomicroscopic images. True dimensions need to be determined for assessing treatments or experimental procedures. In order to determine the true size one also needs to understand the errors in measurements. If the object needs to be measured for analysis then one has to reconstruct the 3D image. This practice is know as stereology, which involves interpreting 2D images from sections of a 3D object to determine its 3D shape or even 3D images and correcting for serial slices being compiled.

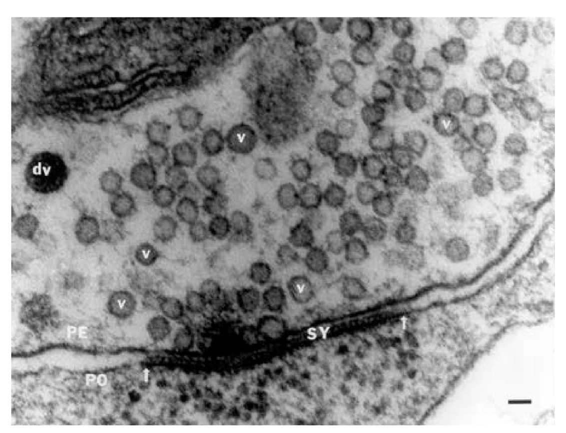
The approach is intended to encourage conceptual thinking and open a range of student-driven explorations at the intersection of biology and mathematics. Doing activities for the sake of hands-on activities in science without a focus on conceptual understanding does not produce effective outcomes on learning science (Windschitl, Thompson, & Braaten, 2008a,b). In addition, the lack of cross-disciplinary training in K-20 grades results in compartmentalization of learning and hampers authentic educational integration of STEM disciplines. This is even evident within mathematics and biology coursework at the early undergraduate levels. We developed interdisciplinary modules that can be used for middle school through college level courses. The modules examine constructing a 3D image of a cell or a synapse within a nerve terminal based on 2D slices of the specimen.

Detailed procedures were developed to accompany each module to guide teachers and students through the process of formulating hypotheses and experimental design, collecting data, analyzing data, and drawing conclusions. Throughout the process, there are a series of questions to serve as a means of formative assessment.

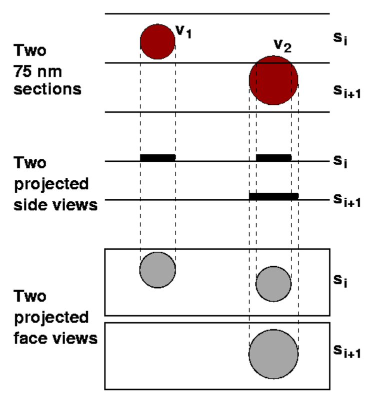
Application of knowledge to real life problems in authentic scientific inquiry with active learning process along with construction of various types of models is a focus for the Next Generation Science Standards. In addition, applying multiple disciplines and cross-cutting concepts integrating algebra and geometry in relation to stereological issues is of importance in appreciation of mathematical application to biological topics.

One can create various scenarios for a classroom to work on:

1. What are the real sizes of synaptic vesicles in nerve terminals (electron microscopic images)?
2. What is the area of a synapse within a nerve terminal (electron microscopic images)?
3. What is the volume of a tumor?
4. How many serial sections would be required to have less than a 10% error in true area or volume of the object?



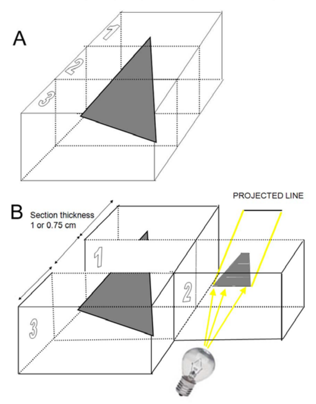
Electron microscopic image (EM) of a synapse (SY) with vesicles (V). Presynaptic nerve terminal (PE); Postsynaptic nerve terminal (PO)

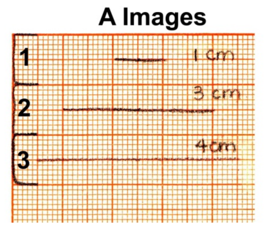


Spheres of two different sizes can project their dimensions similarly when one is only partially sectioned. An error in assuming true dimensions needs to be considered for such problems.

METHODS

A class could have a set of objects for the students to use or provide them with the line drawings on a graph paper. To view the sections, you shine a light from the side of the sections to view a shadow on a 2-D plane (Figure A and B). Figure A shows a flat triangle in the box that will be sectioned. Figure B shows a slice (#2 section) taken out of the box and a light projecting the image on to the wall behind.





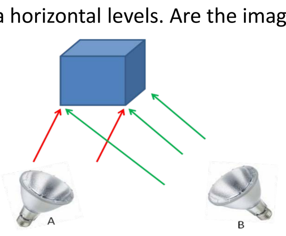
The materials need are all obtained at a local craft store or retail outlet like Wal-Mart.

- Clear Tupperware containers. Allow light to shine through the bottom
- Styrofoam
- Exacto knife
- Super glue
- Graph paper
- Flashlight
- Pens or markers
- Module Worksheet

Sample pre-test questions for assessment:

1. A man is holding a ball in the air and a light source is shined on the ball at a horizontal level to the ball. What shape do you expect the ball's shadow to form on the wall?

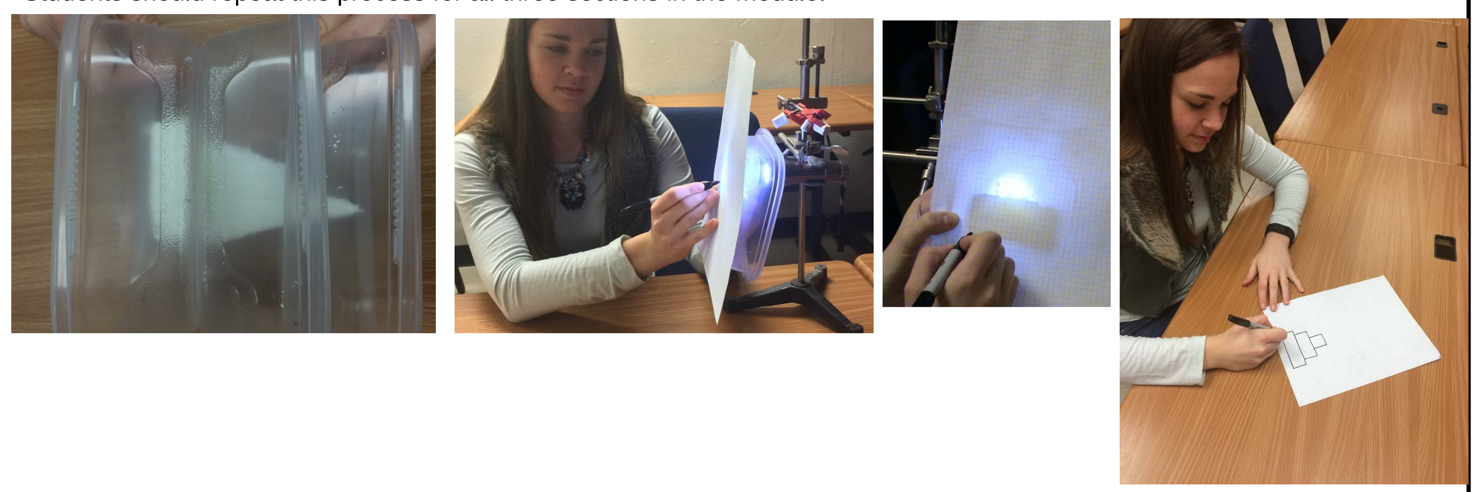
2. Below light A is directly facing one side of the box, but light B is facing the corner of the box, and both are the same distance from the box. The box is 1 cm on every side. Draw the estimated image projected for light A and light B at a horizontal levels. Are the images the same size?



Module 1: Middle School

For this exercise we suggest using an example with a triangular structure. One will first need to a cut a triangle that has a length spanning three small sandwich size Tupperware-like containers. Now cut this triangle into thirds so that a section can be glued into each container. Using glue or strong double stick tape secure a section into the bottom of each container, aligning them so that when the containers are set end to end one can see the triangle shape from above (**Figure 1**).

To use this module, have students pair up. Place a sheet of graph paper on the bottom of the Tupperware container. Now place the container on its side. Another student can shine a flashlight on the open end of the container so that a shadow appears on the graph paper. The student holding the graph paper should trace the shape they see onto the paper. The length measurement should be placed in a table so it will be easy to compare results in the various steps within the exercise. Students should repeat this process for all three sections in the module.



Next, students will calculate different versions of the estimated areas for each object observed in each section. The students can combine the images on the graph paper and try to determine the full structure with these serial sections.

At this point, students will likely come up with various potential objects on a graph paper that the structure could be. This can be used as a discussion topic in a classroom and rationale for their predictions. If the students have not addressed errors in their measurements, guided inquiry by the instructor could introduce questions for the students to work out while not providing an answer but promoting further discussion by the students. Then the students can start to deduce various means of solving the introduced problems with various approaches.

The tabular worksheet will help the students to determine for themselves that even the best approximation to the area of the triangle is not accurate to the actual measurement of the triangle. Despite the best approximations, one still may not obtain the actual area using these sliced specimens as illustrated.

Part 2.0: Determining the Area with an Edited Version of the End Sections

To address better approximations of the true area, the instructor may have to introduce best estimates for structures within the end sections. By assuming on average many trials of taking the same object and slicing it into three sections, the average end locations would end up only projecting half the width into the two end sections. So now the students estimate the area if the end images only projected half the width.

To draw the edited version, find the middle point between sections 1 and 2 and draw 2 vertical lines (C). Draw this dotted line C halfway between the first section to the second section. Line C will stop at the beginning of section 2 and line B will be the estimated end of the specimen. The last section can not have the stair-step effect because if there is no next section that is smaller than the original projected section.

Figure 1.0 A

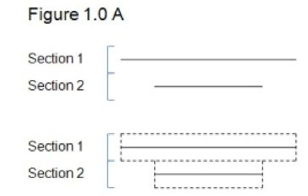
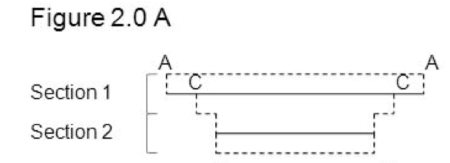


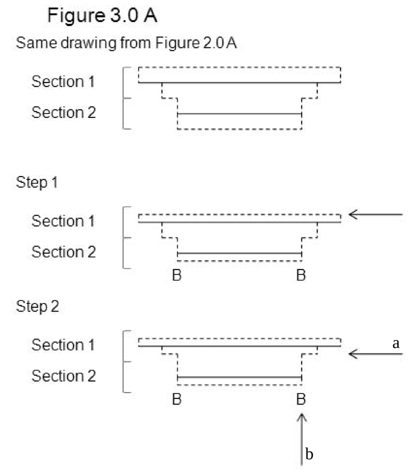
Figure 2.0 A



Part 3.0: Determining the Area with a More Realistic Edited Version of the End Sections

Take the same drawing done in part 2.0 (Figure 2.0 A) and take off half of the width in the end sections (step 1). Then draw a line down a fourth (1/4) of the way from projected Section 1 (step 2, line a), and go to the edge of the projected Second 2 (step 2, line b). This corrects for end sections as well as the large stair step effect on calculating the average area.

Figure 3.0 A



Wrap up discussion:

1. Have students compare the areas with and without end error corrections and the edge corrections.

2. Have students try to determine the real structure and use geometry to calculate the true area for comparisons.

3. Post test

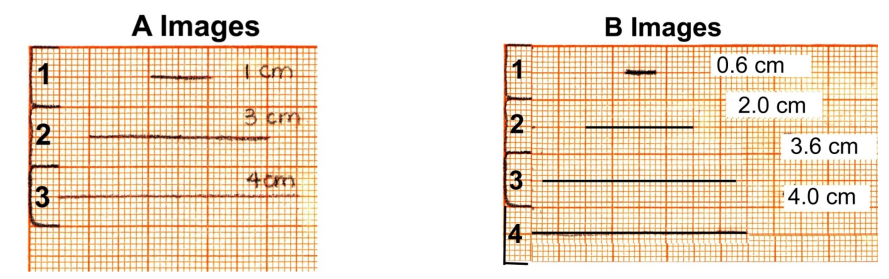
Module 2: High School / College

The students could complete Part 1 without building a model if they were provided the dimensions of the lines from the three sections and guided to estimate the surface area of the slides object. This way they would not have to build the model to work through the exercise. To proceed to Part 2, it would be beneficial for classroom discourse if students were provided a triangular prism to move around in one's hand and to section into three pieces.

With the shadow box approach as mentioned in Part 1, instead of just drawing a line on the graph paper, the height of the object now becomes important in estimating volume of the object.

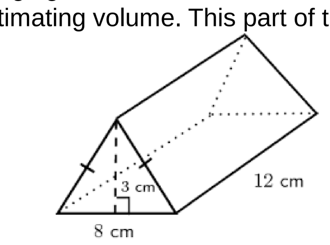
As for a more advanced aspect to determining area for a pre-calculus class, the surface area of the potential objects could be estimated by use of fine grid graph paper and using Simpson's Rule using the same principles of slicing the object into 3 sections as in Part 1.

Another option could be having the given object sliced into **4 or 6 slices**. Then the estimated area, with the error for the end sections, can be compared between the two methods (3 slices versus 4 or 6 slices).

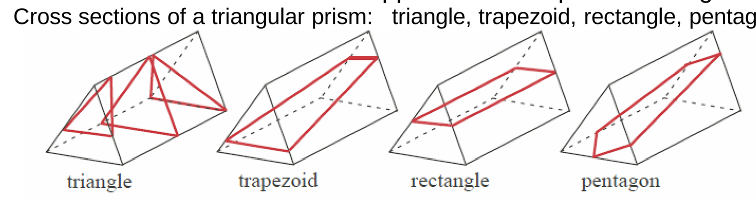


If the estimation of the **errors in the edges between sections** was not addressed in Part 1 but in Part 2, this could be a more advanced aspect of the module. However, guided injury in how to go about estimating a gradual (as compared to a linear change) between edges of adjoin sections maybe have to be explained by the instructor. The approach can be estimates in half steps in length changes from each section and only progressing through half of each section so that it is a gradual stair step change.

For more of a challenge, the **volume of the triangular prism** and the error in volume can be tackled. This can be related to 3-D imaging in MRI machines and determining the level of section thickness based on the size of the object with an acceptable error in estimating volume. This part of the module could also be used to discuss what happens with oblique sectioning.

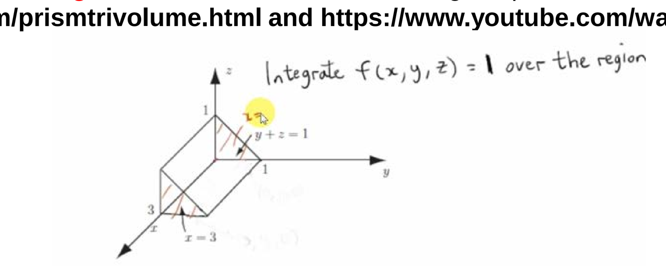


Cross sections of a triangular prism: triangle, trapezoid, rectangle, pentagon



The **volume of the triangular prism** can be approached by using geometry **(Volume=½ length*width*height)** for a true measure. This can be compared to the estimated measures obtained by the projected images and taking the various errors in measurement for the end sections and the in-between sections.

For **calculus based students** a triple integral can be used to determine volume and be compared to the geometric approach as well as to the estimates from the projected lines with thickness. There are various movies posted on YouTube to help explain the steps of how to set up the **triple integral** to calculate volume of a triangular prism:
http://www.mathopenref.com/prismtrivolume.html and https://www.youtube.com/watch?v=E9itLQAoPdE



$$V = \iiint dV = \int_{z=0}^{1} \int_{y=0}^{1-z} \int_{x=0}^{3} 1 \, dx\,dy\,dz$$

Analyzing obtained data for part 2 would be best to have worksheets with the paradigm used (3 or 4-6 slices, with or without error for ends and edges). Separate worksheets for area and volume would help the students focus on the tasks.

Use of computer software

In conjunction with the modules developed, we have also developed computer simulations that demonstrate the same principles. Using **sketch up modeling software**, students can run a simulation that correspond to the module. Since this is a free software, the computer simulations are both cheaper and faster to implement than building physical models. The computer simulations do, however, remove some of the hands on factor that would appeal to and interest many students.

Drawing conclusions and student assessment

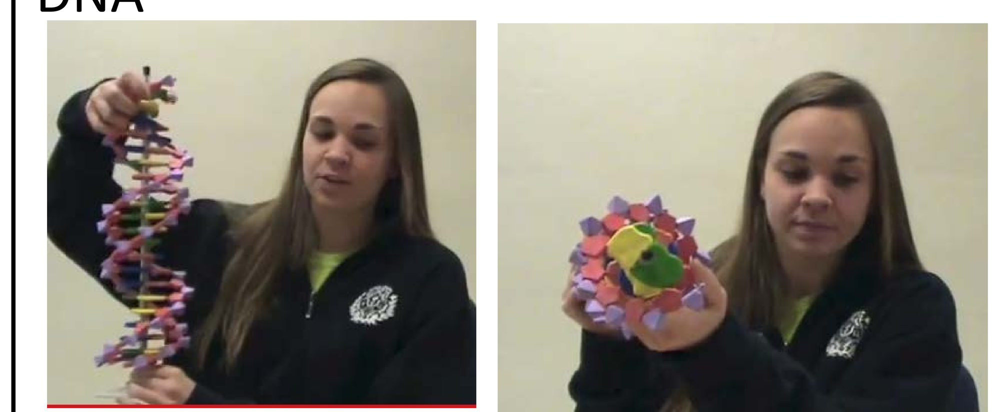
The data collected on the worksheets would be helpful for the teachers to see the students work and thought process into the concepts presented. The worksheet would also be good for classroom discussion into the various means in estimating area and volume of an object by 2D slices projected as compared to already knowing the size and shape of an object. Real life examples of a tumor being treated by a new compound and determining if the treatments is working would be an engaging exercise for students.

NGSS Standards

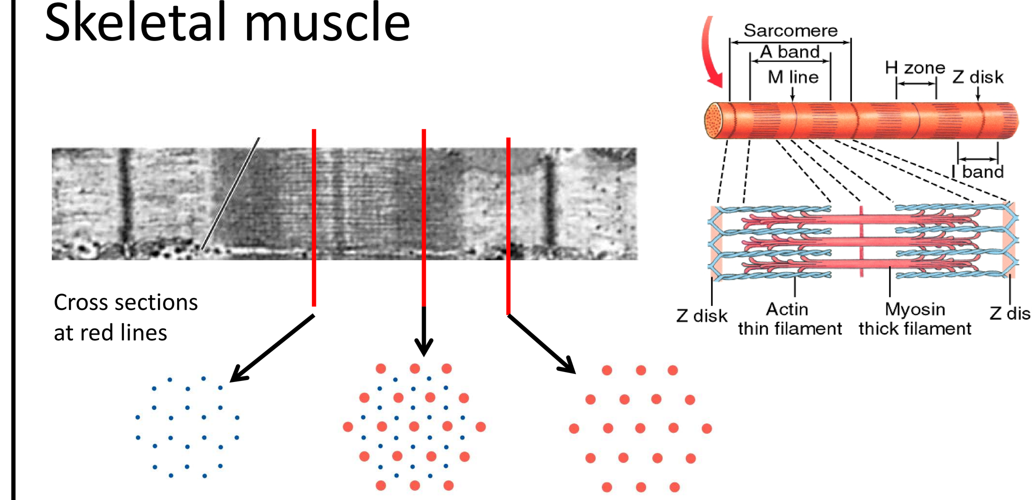
K-2-ETS1-2 Engineering Design
K-2-ETS1 Engineering Design
MS-LS1 From Molecules to Organisms: Structures and Processes
MS-LS1-2 From Molecules to Organisms: Structures and Processes
HS-LS1 From Molecules to Organisms: Structures and Processes
HS-LS1-2 From Molecules to Organisms: Structures and Processes

Applications in which 2D images helped to understand function and form:

DNA



Skeletal muscle



Students try to determine 3D structures from cross sections

Conclusions

1. We developed a module addressing stereological issues which relates to real world problems. This maybe be engaging to students and teachers.

2. Modules examining 2D image projections for reconstruction of 3D objects allows a student to develop various approaches on their own to see 1st hand the problems in the real world. With literature searching in scientific journals and resources, students will be exposed to new concepts and ideas.

3. Determining the best way to process 2D images for 3D reconstructions provides the students with integration of algebra, geometry, and calculus to biological problems.

4. Computer simulations allow teachers and students to quickly collect the data necessary to draw accurate conclusions. The models are faster and cheaper to implement, but students may struggle to understand what is happening behind the scenes to cause those results.

5. In the future, this procedure can be adapted to address additional factors that influence imaging topics such as oblique sections of the objects. The process of adapting Netlogo models to describe biological processes can also be applied to other areas of interest.

References

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